647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov.- 2024 (Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any one out of two. (07)

1. Prove that a map $d: X \times X \to \mathbb{R}$ defined by $d(x,y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$; $\forall\, x, y \in X$ is a metric on a non-empty set X.
2. State and prove necessary and sufficient condition for open set in metric space.

Que.1(B) Answer any one out of two. (04)

1. Find the δ-neighbourhood of a point of a discrete metric space.
2. Prove that a neighbourhood of a point of a metric space is an open set.

Que.1(C) Answer any one out of two. (03)

1. Prove that the finite subset of a metric space is closed.
2. Find all interior points of the set $\{1,2\}$ of the discrete metric space $\mathbb{R}$.

Que.2(A) Attempt any one out of two. (07)

1. State and prove necessary and sufficient condition for a bounded function f on [a,b] to be R-integrable.
2. If a bounded function $f \in R[a,b]$ then $|f| \in R[a,b]$ and $\left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx$

Que.2(B) Attempt any one out of two. (04)

1. Find $\int_0^1 x^2\, dx$ by first definition of Riemann integration.
2. If f is continuous on $[a,b]$ then $f \in R[a,b]$.

Que.2(C) Attempt any one out of two. (03)

1. Prove that $\underline{\int_a^b} f(x)dx \leq \overline{\int_a^b} f(x)dx$
2. State Darboux theorem.

Que.3(A) Attempt any one out of two. (07)

1. State and prove fundamental theorem of integration.
2. Show that G is a commutative group if $(ab)^i = a^i b^i$; $\forall\, a, b \in G$ for any three consecutive integers.

Que.3(B) Attempt any one out of two. (04)

1. State and prove first mean value theorem of integral calculus.
2. Check whether $*$ is a binary operation or not : $a * b = a + b - ab$; $\forall\, a, b \in \mathbb{R} - \{1\}$

Que.3(C) Attempt any one out two. (03)

1. Evaluate: $\lim_{n\to\infty} n \sum_{k=1}^{n} \frac{1}{n^2+k^2}$
2. Prove that every element of a finite group is of finite order.

Que.4(A) Attempt any one out of two. (07)

1. State and prove Lagrange's theorem for a Finite Group.
2. Prove that the composition of two disjoint cycles in S_n is commutative.

Que.4(B) Attempt any one out of two. (04)

1. Prove that the intersection of two subgroups of a group is again a subgroup.
2. Let G be a finite group and let $a \in G$ be a fixed element of G. Then prove that $f_a: G \to G$ defined as $f_a(x) = xa$; $\forall x \in G$ is a permutation on G.

Que.4(C) Attempt any one out of two. (03)

1. Let $H \leq G$ and let $a, b \in G$. Prove : $a \in H$ iff $H = H_a$.
2. Check whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ is even or odd.

Que.5(A) Attempt any one out of two. (07)

1. A subgroup H of a group G is a normal subgroup of G iff $aHa^{-1} \subseteq H;\ \forall a \in G$.
2. Prove that an infinite cyclic group has exactly two generators.

Que.5(B) Attempt any one out of two. (04)

1. Show that $(\mathbb{R}_+, \bullet) \cong (\mathbb{R}, +)$
2. State and prove Cayley's theorem.

Que.5(C) Attempt any one two. (03)

1. Check whether $S_3\ and\ \mathbb{Z}_6$ are isomorphic or not.
2. Define Automorphism.

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